Distributional Effects of Aggregate Shocks: Functional vs. Panel Approaches

Stephanie Ettmeier[1] Chi Hyun Kim[1] Frank Schorfheide[2]

[1]University of Bonn

[2]University of Pennsylvania, CEPR, NBER, PIER

May 20, 2024

Questions and Empirical Strategies in the Existing Literature

Q1: What is the effect of an aggregate shock on the cross-sectional distribution of x?
Q2: How does the x of particular households (or firms) respond to an aggregate shock?

	Approach	Data Requirements	Examples
Q1	**Functional VAR**	Rep. cross sections	Chang, Chen, Schorfheide (2024); Chang, Schorfheide (2024); Ettmeier (2023)
Q1	VAR with inequality stats	Rep. cross sections	Coibion, Gorodnichenko, Kueng, Silvia (2017); Furceri, Loungani, Zdzienicka (2018); Guerello (2018)
Q1, Q2	Indirect calculation: multiply income component or asset share by aggregate IRF	One cross section	McKay, Wolf (2023); Lenza, Slacalek (2023); Del Canto, Grigsby, Qian, Walsh (2023)
Q2	**Panel model to track individuals** (usually group heterogeneity)	Panel data (admin)	Holm, Paul, Tischbirek (2021); Almuzara and Sancibrian (2023); Amberg, Jansson, Klein, Rogantini Picco (2022); Andersen, Johannesen, Jorgesen, Peydro (2021)
Q2	Pseudo panel to track groups	Rep. cross section	Anderson, Inoue, Rossi (2016); Cloyne, Ferreira, Surico (2020); Mitman, Broer, Kramer (2022)

Overview / Contribution

- Administrative data set from Germany that has panel structure but can also be used as repeated cross sections.
- **Empirical contribution 1:** estimate a functional VAR (building on our earlier work) to measure the response of the earnings distribution to a productivity shock for Germany.
- **Methodological contribution:** replace the functional part of the VAR by unit-level income dynamics equation (panel+VAR), discuss model features, outline estimation strategy.
- **Empirical contribution 2:** compare the panel+VAR responses to the functional VAR results.
- Discuss pros and cons of the respective empirical approaches.

Overview

1. **Functional vs. panel modeling**
2. Data set used in this project
3. Functional model: specification and empirical results
4. VAR + panel model: specification and empirical results
5. Conclusion

Stylized Example: Interaction Between Macro- and Microdynamics

- Aggregate variable y_t, cross-sectional variable x_{it} with density $p_t^x(x)$.
- Macro dynamics:

$$y_t = B_{yy} y_{t-1} + \int B_{yl}(\tilde{x})[\ln p_{t-1}^x(\tilde{x})]d\tilde{x} + \epsilon_t, \quad \epsilon_t \overset{iid}{\sim} p_\epsilon(\epsilon). \tag{1}$$

- Individual-level dynamics:

$$x_{it} = \lambda_{i1} y_t + \lambda_{i2} y_{t-1} + \phi_{xx} x_{it-1} + \eta_{it}, \quad \eta_{it} \overset{iid}{\sim} p_\eta(\eta), \quad (\lambda_{i1}, \lambda_{i2}) \overset{iid}{\sim} p_\lambda(\lambda_1, \lambda_2). \tag{2}$$

- Density (functional) dynamics (can be linearized):

$$p_t^x(x) = \int \int p_\eta\big(x - \lambda_1 y_t - \lambda_2 y_{t-1} - \phi_{xx}\bar{x}\big) p_\lambda(\lambda_1, \lambda_2) p_{t-1}^x(\bar{x}) d(\lambda_1, \lambda_2) d\bar{x}. \tag{3}$$

- **Functional VAR approach:** estimate (1) and of (3) linearized wrt. $\ell_t(x) = \ln p_t^x(x)$.
- **Panel approach:** estimate (1) and (2).

Overview

1. Functional vs. panel modeling
2. **Data set used in this project**
3. Functional model: specification and empirical results
4. VAR + panel model: specification and empirical results
5. Conclusion

German Administrative Data: SIAB

- Panel data set containing a 2% sample of all individuals ever registered in the social security system.
- Covers $\approx$ 80% of German labor force: excludes self-employed and civil servants.
- Data on daily earnings, together with working days per spell.
- Earnings are top-coded at the social security contribution ceiling.
- Sample selection: 1992:Q1 - 2019:Q4.

Micro-level Observables

- Employment status: $s_{it} \in \{1(E, \text{employed}), 2(U, \text{unemployed}), 3(O, \text{out of sample})\}$.
- Assume that number of units i in the (E,U,O) universe is constant
  by moving units into O state when they drop out of sample.
- Observe labor earnings when working: $\tilde{x}_{it}\mathbb{I}\{s_{it} = 1\}$.
- Average cross-sectional earnings

$$\bar{x}_t = \frac{\sum_{i=1}^{N} \tilde{x}_{it}\mathbb{I}\{s_{it} = 1\}}{\sum_{i=1}^{N} \mathbb{I}\{s_{it} = 1\}}. \tag{4}$$

- Standardization + inverse hyperbolic sine transformation of observed earnings **to remove trend and capture spatial correlation due to aggregate shocks**:

$$x_{it} = f(\tilde{x}_{it}/\bar{x}_t). \tag{5}$$

- Unemployment rate

$$UR_t = \frac{\sum_{i=1}^{N} \mathbb{I}\{s_{it} = 2\}}{\sum_{i=1}^{N} \mathbb{I}\{s_{it} = 1\} + \sum_{i=1}^{N} \mathbb{I}\{s_{it} = 2\}}. \tag{6}$$

Aggregate Data and Shocks

- **Macro variables:**
  - Log labor productivity, measured as total hours worked/total GDP (Federal Statistical Office Germany)
  - Log real GDP per capita (Federal Statistical Office Germany)
  - Log average earnings (SIAB) $\ln \bar{x}_t$ from above
  - Unemployment rate *or* EE EO UU UO OE OO transition probabilities (SIAB)
- **Recursive shock identification:** shock to labor productivity which is ordered first.
- **Note:** analysis can be done with monetary or fiscal shocks, but we wanted to maximize variation generated by shock.

Overview

1. Functional vs. panel modeling
2. Data set used in this project
3. **Functional model: specification and empirical results**
4. VAR + panel model: specification and empirical results
5. Conclusion

Functional Model for Empirical Analysis

- Each t: econometrician observes Y_t and a sample of N *iid* draws x_{it} from $p_t(x)$.
- Measurement equation for micro data:

$$x_{it} \sim p_t(x) = \frac{\exp\{\ell_t(x)\}}{\int \exp\{\ell_t(x)\}dx}.$$

- State-transition equations:

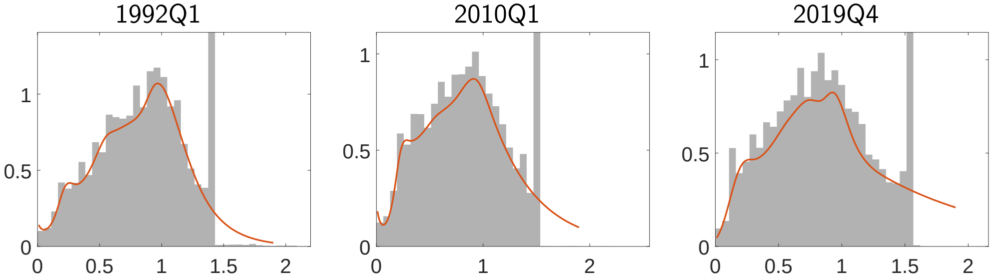
$$\begin{aligned} Y_t &= B_{y0} + B_{yy}Y_{t-1} + \boldsymbol{B}_{yl}[\ell_{t-1}] + u_{y,t} \\ \ell_t(x) &= B_{l0}(x) + B_{ly}(x)Y_{t-1} + \boldsymbol{B}_{ll}[\ell_{t-1}](x) + u_{l,t}(x), \\ & \text{e.g. } \boldsymbol{B}_{ll}[\ell_{t-1}](x) = \int B_{ll}(x,\tilde{x})\ell_{t-1}(\tilde{x})d\tilde{x} \end{aligned}$$

- Use a sieve approximation for $\ell_t(x)$ (and operators) to obtain K-dim model:

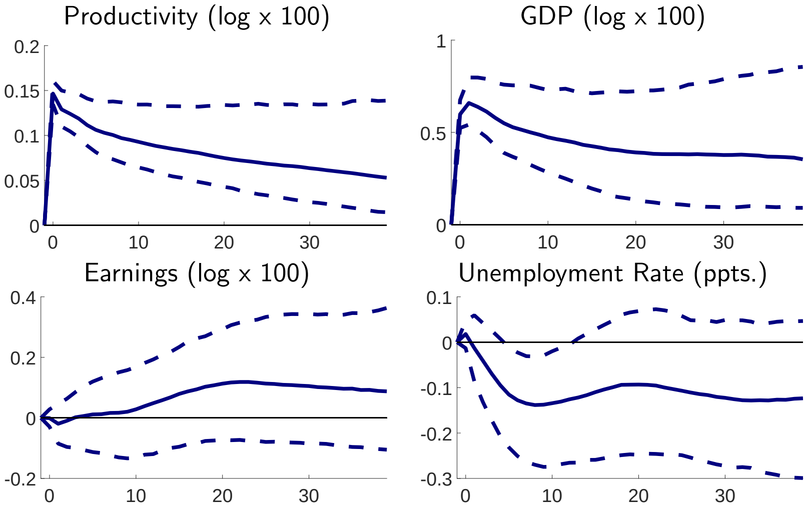
$$\ell_t(x) \approx \ell_t^{(K)}(x) = \sum_{k=1}^{K} \alpha_{k,t}\zeta_k(x) = \zeta'(x)\alpha_t.$$

- **Bayesian estimation:** see Chang, Chen, and Schorfheide (forthcoming, JPE).

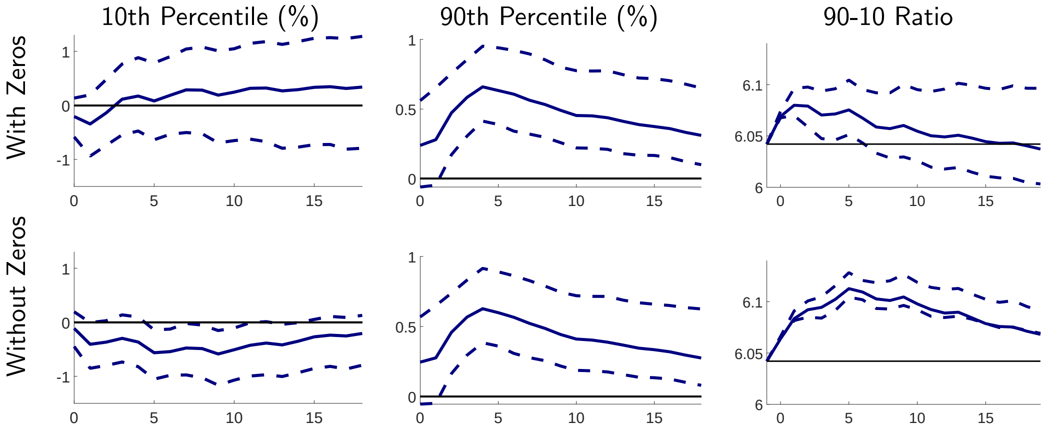
Estimated Densities for Three Time Periods



IRFs of the Aggregate Variables



IRF of Percentiles and Inequality Statistics



Summary of FVAR Results / Next Steps

Summary of FVAR Results:

- Including zero earnings in pctls: unemployment $\downarrow \mapsto$ more units with positive earnings $\mapsto$ 10th percentile $\uparrow$; gains at 90th percentile are stronger $\mapsto$ inequality $\uparrow$.
- Excluding zero earnings in pctls: drop at 10th percentile could be result of rigid labor market for low-skilleds and slowly adjusting wages.
- **Cyclicality of inequality:** pro-cyclical in German data; countercyclical in U.S. data.

Next Steps:

- Keep the aggregate VAR part but replace the functional dynamics in previous model with unit level x_{it} dynamics.
- Transition probabilities for employment status (E, U, O) replace unemployment rate in vector of aggregate variables.
- Generate IRFs at unit level and then aggregate into distributional response.

Overview

1. Functional vs. panel modeling
2. Data set used in this project
3. Functional model: specification and empirical results
4. **VAR + panel model: specification and empirical results**
5. Conclusion

VAR+Panel Model

- Y_t stacks three macro observables:
  labor productivity, real GDP per capita, log average earnings.

- Define unobserved transition probabilities: $\Pi_{jk,t} = \mathbb{P}\{s_{it} = j | s_{it-1} = k\}$.
- $\mathcal{D}_{1:N,1:T}$ collects the micro observables $(s_{it}, x_{it}\mathbb{I}\{s_{it} = 1\})$ for $i = 1, \dots, N$ and $t = 1, \dots, T$.
- For $p = 1$ we obtain factorization:

$$
\begin{aligned}
& p(Y_{1:T}, \Pi_{1:T}, \mathcal{D}_{1:N,1:T} | \theta) \qquad (7) \\
& = \prod_{t=1}^{T} \Bigg(\underbrace{p(Y_t, \Pi_t | Y_{t-1}, \Pi_{t-1}, \ell_{t-1}, \theta) p(\ell_{t-1} | \mathcal{D}_{it-1})}_{\text{VAR part}} \\
& \qquad \underbrace{\times\, p(s_{1:N,t} | \Pi_t, \mathcal{D}_{it-1}) \prod_{i=1}^{N} p(x_{it}\mathbb{I}\{s_{it} = 1\} | s_{it}, Y_t, Y_{t-1}, \mathcal{D}_{it-1}, \theta)}_{\text{panel part}} \Bigg).
\end{aligned}
$$

VAR+Panel Model: Some Remarks

- Typically panel and VAR models are not integrated. **We do integrate the two components.**
- Large sample arguments are required to
  - replace latent $\Pi_{kt,t}$s by estimates $\hat{\Pi}_{jk,t} = N_{jk,t} / \sum_{j=1}^{3} N_{jk,t}$;
  - replace ℓ_{t-1} by $\hat{\ell}_{t-1}(\mathcal{D}_{it-1})$.
- Model and parameters are set up so panel and VAR part can be estimated separately.
- We assumed that transition probabilities for s_{it} do not depend on unit-level characteristics. Abstract from selection effects for now.
- "Missing intercept" (McKay and Wolf; Barnichon and Mesters) implicitly through feedback from lagged cross-sectional distribution into aggregate variables (not yet implemented).

VAR+Panel Model: Three-Step Estimation

1. $\hat{\Pi}_t$ from transition counts.
2. θ_{agg} based on VAR with $\hat{\Upsilon}_t = [Y_t, \hat{\Pi}_t]$ with $\hat{\ell}_{t-1}(x)$ as additional explanatory variable (not yet implemented for current results).
3. **θ_{mic} based on panel model which includes:**
   - $U \mapsto E$ transitions: $p(x_{it}\mathbb{I}\{s_{it} = 1\}|s_{it} = 1, Y_t, s_{it-1} = 2, \theta_{mic})$;
   - $O \mapsto E$ transitions: $p(x_{it}\mathbb{I}\{s_{it} = 1\}|s_{it} = 1, Y_t, s_{it-1} = 3, \theta_{mic})$;
   - $E \mapsto E$ transitions: $p(x_{it}\mathbb{I}\{s_{it} = 1\}|s_{it} = 1, Y_t, \mathcal{D}_{it-1}, \theta_{mic})$.

VAR+Panel Model: $E \mapsto E$ Transitions

- (Simplified) earnings process (Z_t is a linear function of (Y_t, Y_{t-1}))

$$x_{it} = \rho x_{it-1} + \alpha_i + \beta_i' Z_t + \eta_{it}, \quad \eta_{it} \sim \mathcal{N}(0, \sigma_i^2). \tag{8}$$

- (Parametric) correlated random effects assumption:

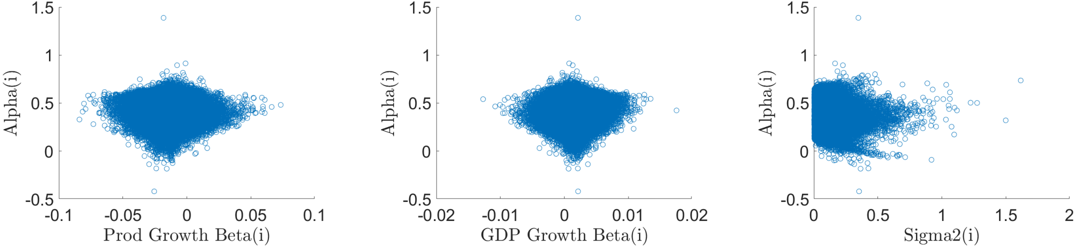
$$(\alpha_i, \beta_i, \sigma_i^2) \overset{iid}{\sim} p(\alpha, \beta | x_{i0}, \sigma^2, \xi) p(\sigma^2 | \xi) \tag{9}$$

$$p(\alpha, \beta | x_{i0}, \sigma^2, \hat{\xi}) \equiv \mathcal{N}\left(\begin{bmatrix} 0.08 + 0.11 x_{i0} \\ -.016 \\ .004 \end{bmatrix}, \sigma^2 \begin{bmatrix} 0.21 & & \\ 0 & \frac{6E-6}{\lambda} & \\ 0 & -\frac{7E-7}{\lambda} & \frac{2E-7}{\lambda} \end{bmatrix} \right), \quad \lambda = .0025$$

$$p(\sigma^2 | \hat{\xi}) \equiv IG\left(\underline{\nu} = 5, \underline{s}^2 = 5 \cdot 0.18^2\right)$$

- **Bayesian estimation...** (in progress). Results are obtained by taking a couple of short cuts.

Scatterplots of Posterior Mean Estimates $\bar{\alpha}_i$ and $\bar{\beta}_i$



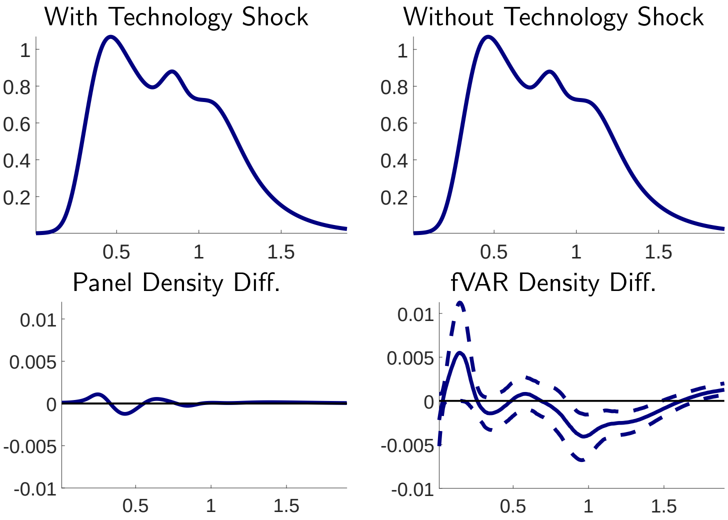
Impulse Response of Cross-Sectional Units + Densities

Fix an event date T_0, e.g., 2010:Q1. Shock happens in Q2.

For $h = 0, \ldots, H$**:** for baseline "0" trajectories (no productivity shock at $h = 0$) and shocked "s" trajectories (one std dev productivity shock at $h = 0$).

1. Using cross-sectional data from period $t = T_0 + h - 1$ estimate coefficients for cross-sectional log densities $\ell^0_{t+h-1}(\cdot)$ and $\ell^s_{t+h-1}(\cdot)$
2. Iterate estimated aggregate VAR forward to obtain Υ^0_{t+h} and Υ^s_{t+h} (based on Υ^0_{t+h-1}, Υ^s_{t+h-1}, ℓ^0_{t+h-1}, and ℓ^s_{t+h-1}).
3. Conditional on Υ^0_{t+h} and Υ^s_{t+h}, generate $(x^0_{it+h}\mathbb{I}\{s^s_{it+h} = 1\}, s^s_{it+h})$ and $(x^s_{it+h}\mathbb{I}\{s^s_{it+h} = 1\}, s^s_{it+h})$ for $i = 1, \ldots, N$.
4. Estimate log spline densities to obtain $p^0_{t+h}(\cdot)$ and $p^s_{t+h}(\cdot)$.

Impulse Response of Cross-sectional Densities, $h = 4$



Remarks

- **Results are very preliminary.**
- Density differentials are qualitatively similar, but quantitatively different.
- Distributional effect obtained from panel analysis is more muted. Modeling of income dynamics needs to be refined in various dimensions (negative income in simulations, top coding, etc.).
- Once there is a match between distributional responses, panel information can be used to shed more insights into explanations for the distributional shifts. (How do certain groups of individuals react?)

How Does All of This Relate to Panel Local Projections?

- In a VAR setting, multi-step regressions can recover coefficient matrices of a Wold representation.
  (+) less bias if VAR alternative is misspecified;
  (-) can be inefficient compared to VAR estimation; see Schorfheide (2005).
- In a panel setting, trade-offs are largely unexplored. Requires a Wold representation for a high-dimensional process with some nonlinearities.
- Panel local projections typically assume observed group heterogeneity

$$x_{it} = \rho_{g_i} x_{it-1} + \alpha_{g_i} + \beta'_{g_i} Z_t + \eta_{it}, \quad g_i \in \{1, 2, \dots, G\}.$$

$\Longrightarrow$ parsimonious, but typically ignores a large amount of heterogeneity
$\Longrightarrow$ OK for Q2, but not good for Q1.

Conclusion

- **Use German administrative data to compare two empirical approaches...**
- Functional modeling:
  - (+) repeated cross-sections suffice
  - (+) unit-level behavior and heterogeneity does not need to be explicitly modeled
  - (−) cannot track behavior of cross-sectional units.
- Panel modeling:
  - (+) ability to track unit-level behavior
  - (−) estimation requires panel data
  - (−) challenging to specify unit-level law-of-motion: heterogeneity, non-Gaussianity, nonlinearity
- In practice, researchers are limited by the availability of data sets. Insights from this research may be useful for combining different types of data sets and conducting analyses with mixed-frequency data.